Abstract—In Wireless multimedia sensor networks (WMSNs), two graphs, communication network graph and vision graph, can be established. The camera nodes connected in the vision graph share overlapped field of views (FOVs) and they depend on the densely deployed relay nodes in the communication network graph to communicate with each other. Given a uniformly deployed camera sensor network with relay nodes, the problem is to find the number of hops for the vision-graph-neighbor-searching messages to construct the vision graph in an energy efficient way. In this paper, mathematical models are developed to analyze the FOV overlap of the camera nodes and the multi-hop communications of WSNs in two dimensional, which are utilized to analyze the optimal hop number. In addition, simulations are conducted to verify our models.

I. INTRODUCTION

Recently, the advances in low cost CMOS imaging sensors have made wireless multimedia sensor networks (WMSNs) possible [1][5]. In WVSNs, multiple camera sensors are deployed to monitor an area. Potential applications of WMSN include surveillance, traffic tracking, wildlife monitoring and battle-field monitoring.

The challenges faced by scalar wireless sensor networks, such as energy constraints, limited processing capabilities, unreliability and low accuracy of obtained data, are only exacerbated in WMSN. To improve the sensing quality, a certain level of collaboration among sensor nodes are required. In [1], collaborative multimedia in-network processing is suggested, which can utilize the computational capacity of sensors as well as reduce communication cost and energy consumption. In [2], scalar sensors are exploited to help camera nodes detect events in WMSNs. More directly, camera sensors can collaborate with each other to accomplish a task. This is achieved by exploiting the overlapped field-of-views of the camera sensors.

In WMSN, each camera sensor has its own directional sensing range, known as the field-of-view (FOV), and cameras may have overlapped FOVs, which means if an event occurs in this overlapped area, several cameras may capture this event in different perspectives. These camera nodes form a vision graph [4], in which an edge between two cameras means they share an overlapped FOV and the two cameras are called vision graph neighbors. Some research exploits the characteristics of the overlapped FOVs in collaboration of camera sensor nodes. In [6], routing paths are established based on the overlapped FOVs of camera nodes. In [3] and [8], overlapped FOV is exploited to define correlation among camera nodes and this correlation is used for cooperative video processing. Meanwhile, in [4], the images of the overlapped FOVs have been explored to calibrate the camera nodes.

To construct vision graphs in WMSN, two methods are widely used in literature: the cameras nodes know their locations and directions a priori [3], or reference objects are used for camera nodes to calculate overlapped FOVs [7][8]. However, in these works, simple flooding is employed to exchange information among camera nodes. It is well known that for large scale WSNs, flooding is impractical in terms of communication cost and network congestion. In this paper, we propose a limited-hop-number multi-hop communication for constructing the vision graph in WMSNs. Given a uniformly deployed camera sensor network with relay nodes, we are interested in finding the hop number for the vision-graph-neighbor-searching messages (hello message) in order to construct the vision graph in an energy efficient way.

Mathematical models are developed to analyze the probability of constructing the vision graph for different message lifetime (maximum hop number). This is achieved by developing a probabilistic model of overlapped FOV for different distance and the probabilistic model of propagation distance for different message life time. In [9], the authors have proposed a method to analyze the probability distribution of multi-hop communications in WSNs. However, only one dimensional topology is considered and the channel is assumed to be perfect. In this paper, we propose a recursive method to analyze the multi-hop communications of WMSNs in two dimensional topologies. Moreover, a more practical channel model is adopted to capture the connectivity probability. In addition, simulations are conducted to verify our models.

To our knowledge, this is the first paper to address the communication problem of constructing vision graph in WMSNs. Our main contributions are:

- The probabilistic model of two cameras having overlapped FOV with respect to their distance;
- The 2-D multi-hop communication model that maps maximum hop number to connectivity;
- The mathematical model to calculate the needed maximum hop number to construct the vision graph for a given WSMN.

The remainder of this paper is organized as follows: The probabilistic models for overlapped FOV, multi-hop communication and vision graph construction are developed in Section II. Simulations and Numerical analysis of those models is provided in Section III. Furthermore, the conclusions are drawn in Section IV.

II. SYSTEM MODEL

We consider applications, in which the camera sensors are randomly deployed. To connect these camera sensors, some other relay nodes are also randomly deployed among them by Poisson random process. The overview of the system is shown in Fig. 1.
In this figure, the two cameras are vision graph neighbors since their FOVs are overlapped. Other relay nodes are provided to help these two camera nodes communicate with each other.

Denote the probability of constructing the vision graph \( p_v \), our goal is to relate this probability to the maximum lifetime of the hello message. \( p_v \) is given as
\[
p_v(n) = \frac{\int_0^\infty p_{\text{enc}}(l, n) \, dl}{\int_0^\infty p_{\text{cn}}(l) \, dl} , \tag{1}
\]
where \( p_{\text{enc}}(l, n) \) is the probability of successfully connected vision graph neighbors given distance \( l \) and maximum hop number \( n \) and \( p_{\text{cn}}(l) \) is the probability of vision graph neighbors given distance \( l \). \( p_{\text{enc}}(l, n) \) is related to the FOV of the camera and it is developed in Section II-A, while \( p_{\text{cn}}(l) \) is related to the multi-hop communication of sensors, whose model is provided in Section II-B. In Section II-C, those two models are utilized to develop the model for vision graph construction.

A. Overlapped Field of View

We model the FOV of a camera as a fan sector in 2D plane, as shown in Fig. 1. It is defined by a tuple \( (r_v, \alpha) \), where \( r_v \) is the FOV radius, which determines the maximum distance the camera can observe, and \( \alpha \) is the visual angle, which depicts the width of the FOV. Two camera nodes are defined as vision graph neighbors if and only if their FOVs overlap.

This overlap model is shown in Fig. 2. It is observed that given the distance \( l \) of two cameras, when these two cameras are at some specific directions, they will have overlapped FOV and become vision graph neighbors. Assume that the direction of the camera is uniformly distributed, the probability that two cameras are vision graph neighbors is equal to the portion of directions at which they have overlapped FOV. When fixing the direction of the first camera at angle \( \theta_1 \), there exists a range \([\theta_{2\min}, \theta_{2\max}]\), such that when the direction of the second camera is in this range, these two cameras have overlapped FOV. As \( \theta_1 \) changes, the range changes too. With the ranges of \( \theta_1 \) and \( \theta_2 \) in which the two camera nodes can have overlapped FOV at distance \( l \), the probability that these two cameras can have overlapped FOV at distance \( l \), which is denote as \( p_o(l) \), is the integral of the ranges divided by the maximum range. Thus,
\[
p_o(l) = \frac{1}{2\pi^2} \sum_1^{\psi} \int_{\psi_1}^{\psi_{1+1}} (\theta_{2\max} - \theta_{2\min}) \, d\theta_1 . \tag{2}
\]

In Table I, the ranges of \( \theta_1 \) and \( \theta_2 \), in which these two camera nodes have overlapped FOV for different distance, \( l \), are listed. For a given \( l \), the range of \( \theta_1 \) is divided into categories, and in each category, the range of \( \theta_2 \) can be expressed by the value of \( \theta_1 \). Note in the tables, the range of \( \theta_1 \) is only considered as in range \([0, \pi]\). For the range \([\pi, 2\pi]\), it is a mirror of \([0, \pi]\). Also note that the camera visual angle \( \alpha \) impacts the conditional probability and in three different cases \([0, \pi], [\pi, 2\pi], [\pi, \pi/2] \), the expressions are different. Here, we assume the visual angle of the camera \( \alpha \) is in the range of \([\pi/2, \pi]\). The variables \( -f \) in the tables are shown in the following.

\[
a = \cos^{-1} \left( \frac{l}{2r_v} \right) \tag{3}
\]
\[
b = \tan^{-1} \left( \frac{r_v \sin(\theta_1 - \alpha)}{l - r_v \cos(\theta_1 - \alpha)} \right) \tag{4}
\]
\[
c = \sin^{-1} \left( \frac{r_v}{l} \right) \tag{5}
\]
\[
d = \cos^{-1} \left\{ \left[ \frac{l}{l \tan^2(\theta_1 - \alpha)} \right. \right.
\]
\[
\left. + \frac{r_v^2 + r_o^2 \tan^2(\theta_1 - \alpha) - l^2 \tan^2(\theta_1 - \alpha)}{r_v(1 + \tan^2(\theta_1 - \alpha))} \right\} \tag{6}
\]
\[
e = \cos^{-1} \left\{ \left[ \frac{l}{l \tan^2(\theta_1 - \alpha)} \right. \right.
\]
\[
\left. - \frac{r_v^2 + r_o^2 \tan^2(\theta_1 - \alpha) - l^2 \tan^2(\theta_1 - \alpha)}{r_v(1 + \tan^2(\theta_1 - \alpha))} \right\} \tag{7}
\]
\[
f = \cos^{-1} \left\{ \left[ \frac{l}{l \tan^2(\theta_1 + \alpha)} \right. \right.
\]
\[
\left. + \frac{r_v^2 + r_o^2 \tan^2(\theta_1 + \alpha) - l^2 \tan^2(\theta_1 + \alpha)}{r_v(1 + \tan^2(\theta_1 + \alpha))} \right\} \tag{8}
\]

As expected, it is noticed from Table I that when the two cameras are close to each other, the ranges in which they can be vision graph neighbors are larger, which implies the probability that they are vision graph neighbors is greater.

B. Multi-hop Communication

In this section, we develop a model to analyze multi-hop communication of two camera nodes in wireless sensor networks. In the analysis, the distance of camera node \( A \) and \( B \) is denoted as \( l \) and the density of the relay nodes is \( \lambda \). In addition, Log-normal shadowing model is employed to represent the channel.

In our model, \( C_l^n \) represents the event that two nodes at distance \( l \) can communicate within \( n \) hops, while \( C_l^n \) represents the opposite. The probability of event \( C_l^n \) is denoted as \( p_{cn}(l) \) and the probability of event \( \bar{C}_l^n \) is \( q_{cn}(l) \). In addition, \( P_{cn}(l) \) is employed
TABLE I: The possible ranges of $\theta_1$ and $\theta_2$ to have overlapped FOV for different distance $l$.

<table>
<thead>
<tr>
<th>$\theta_1$ range</th>
<th>$\theta_2$ range</th>
</tr>
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<tbody>
<tr>
<td>$[0, \alpha - \cos^{-1} \frac{3b}{2r_0}]$</td>
<td>$[\pi - a - \alpha, \pi + a + \alpha]$</td>
</tr>
<tr>
<td>$[\alpha - \cos^{-1} \frac{3b}{2r_0}, \alpha]$</td>
<td>$[\pi - a - \alpha, \pi - b + \alpha]$</td>
</tr>
<tr>
<td>$[\alpha, \alpha - \cos^{-1} \frac{3b}{2r_0} + \alpha]$</td>
<td>$[\pi - a - \alpha, \pi - b + \alpha + \alpha]$</td>
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$r_0 < l \leq 2r_0$

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$-2r_0 \cos(2\alpha) < l \leq r$

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</tr>
<tr>
<td>$[\cos^{-1} \frac{3b}{2r_0} + \alpha, \alpha - \cos^{-1} \frac{3b}{2r_0} + \alpha]$</td>
<td>$[\pi - b - \alpha, \pi + \alpha]$</td>
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<tr>
<td>$[\pi - \alpha, \alpha + \pi]$</td>
<td>$[\pi - e - \alpha, f + \pi + \alpha]$</td>
</tr>
<tr>
<td>$[\alpha + \pi, \pi]$</td>
<td>$[\pi - d - \alpha, f + \pi + \alpha]$</td>
</tr>
</tbody>
</table>

$0 < l \leq -2r_0 \cos(2\alpha)$

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The symbol error rate, $p_s(X, l)$, is calculated by

$$p_s(X, l) = Q(\beta_2(\phi(X, l) - \beta_1)),$$

where $\beta_1$ and $\beta_2$ are two parameters obtained by experiments [11]. Note that other error rate models can be used in our scheme. For example, a means to calculate the error rate for MicaZ mote is developed in [10]. For one hop communication, the probability that two nodes can communicate:

$$p_{c1}(l) = p\{C_1^1\} = \int_{-\infty}^{\infty} (1 - p_0(N, l)) L f_{X_\sigma}(X_\sigma) dX_\sigma,$$

where $L$ is the packet length in symbols, $f_{X_\sigma}(X_\sigma)$ is the PDF of the shadowing effect modeled by a log-normal random variable.

For the one hop situation, the probability that two nodes can communicate within up to 1 hop is given simply by

$$P_{c1}(l) = p_{c1}(l).$$

When the hop number is greater than 2, we develop our model as follows. The event that node $A$ has a $n$-hop communication path to node $B$ equals to that there is node $x$ at position $(\rho_x, \theta_x)$, which has a 1-hop path to $A$ a $(n-1)$-hop communication path to node $B$. The model for this problem is shown in Fig. 3. Assume

The location of the intermediate node, $x$, is $(\rho_x, \theta_x)$, where $\rho_x$ is the distance of $x$ and $B$, and $\theta_x$ is the angle of $x$ with respect to vector $BA$. Thus, the distance of $x$ and $A$ can be expressed as

$$\rho_{xA} = \sqrt{(l - \rho_x \cos \theta_x)^2 + (\rho_x \sin \theta_x)^2}.$$  

Given Poisson process density $\lambda$, the probability that at position $(\rho_x, \theta_x)$ there is a node is

$$p_c(\rho_x, \theta_x) = 1 - e^{-\lambda \rho_x \Delta \rho_x \Delta \theta_x},$$

$$\approx \lambda \rho_x \Delta \rho_x \Delta \theta_x.$$  

The approximation holds when the area $\rho_x \Delta \rho_x \Delta \theta_x \to 0$. The ranges of $\rho_x$ and $\theta_x$ are

$$0 < \rho_x \leq \infty \quad \text{and} \quad 0 \leq \theta_x \leq 2\pi.$$

The probability of 1-hop communication between $x$ and $A$ is $p_{c1}(\rho_{xA})$, where $\rho_{xA}$ is the distance from $A$ to $x$. Recursively, the probability that node $x$ has a $(n-1)$-hop communication path to node $B$ is $p_{c(n-1)}(\rho_x)$. Therefore,

$$p_{cn}(l) = 1 - \prod_{\rho_x \theta_x} (1 - (\lambda \rho_x \Delta \rho_x \Delta \theta_x) p_{c1}(\rho_{xA}) p_{c(n-1)}(\rho_x)).$$

In (16), the term inside the product defines the probability that there is no intermediate node at $(\rho_x, \theta_x)$ which satisfies the requirements. Since the relay nodes are deployed independently, the product over $\rho_x$ and $\theta_x$ is the probability that there is no intermediate node at the whole planar. Finally, subtracting that probability from 1 is the probability that those two nodes can have a $n$-hop communication.

Also, the probability that two nodes at distance $l$ can communicate in up to $n$ hops is

$$P_{cn}(l) = p\{C_1^1\} + p\{C_1^1, C_2^1\} + \ldots + p\{C_1^1, C_2^1, \ldots, C_{n-1}^1\}. $$

The first term in (17) is the probability of communication in 1 hop, the second term is the probability that the two camera nodes cannot communicate in 1 hop but can communicate in 2 hops. Generally, the $n$th term is the probability that the two camera nodes cannot communicate within up to $(n-1)$ hops but can communicate in $n$ hops. The sum is the probability that the two
nodes can communicate within up to \( n \) hops. Since given distance, \( l, C^1, \ldots, C^n \) and \( C^1_i, \ldots, C^n_i \) are independent, we have

\[
P_{cn}(l) = p\{C^1\} + p\{C^1\} p\{C^2\} + \cdots + p\{C^1\} \cdots p\{C^{(n-1)}\} p\{C^n\} = p_{c1}(l) + q_{c1}(l)p_{c2}(l) + \cdots + \left(\prod_{i=1}^{n-1} q_{ci}(l)\right) p_{cn}(l) .
\]

(18)

C. Vision Graph Construction

The short range communication character of WSNs causes a camera node’s vision graph neighbors may be several hops away. On the other hand, two closely located camera nodes may not be vision graph neighbors. Given a uniformly deployed camera sensor network with relay sensors, we try to find the optimal hop number for the hello messages in order to construct the vision graph.

Assume the camera node deployment is a Poisson point process and denote the camera node density as \( \kappa \). Given the distance \( l \), the probability that a camera node has vision graph neighbors at distance \( l \) is

\[
p_{vn}(l) = (1 - e^{-\kappa A_\mu}) p_o(l) \approx \kappa A_\mu p_o(l),
\]

(19)

where \( p_o(l) \) is the probability that two cameras have overlapped FOV when the distance is \( l \), which is provided in Section II-A, and \( A_\mu \) is the size of an infinitely small circle area, which is expressed as

\[
A(\mu) = 2\pi l dl \to 0.
\]

The paths to some far away vision graph neighbors may not be established because of the limitation of the message hop number. Thus, we consider the probability of constructing the vision graph as the ratio of the connected vision graph neighbors against all vision graph neighbors. Given a camera node \( A \), the probability that there are camera nodes at distance \( l \) which are vision graph neighbors of \( A \) and they can communicate within \( n \)-hops is

\[
p_{vnc}(l, n) = p_{vn}(l) P_{cn}(l),
\]

(21)

where \( p_{vn}(l) \) is given in (19) and \( P_{cn} \) is given in (18).

The probability of constructing the vision graph using \( n \)-hop communication is

\[
p_v(n) = \int_0^\infty p_{vnc}(l, n) dl = \int_0^\infty 2\pi l p_o(l) P_{cn}(l) dl = \int_0^\infty 2\pi l p_o(l) p_{cn}(l) dl .
\]

(22)

which can be further simplified by considering the fact that when the distance of two cameras is greater than twice the FOV range \( r_v \), their FOVs do not overlap. Hence, it is impossible that they are vision graph neighbors. Therefore, the upper limit of the integral can be rewritten to \( 2r_v \), thus,

\[
p_v(n) = \int_0^{2r_v} p_o(l) P_{cn}(l) dl .
\]

(23)

III. NUMERICAL ANALYSIS

In this section, numerical analysis of the system model is performed using Matlab. We first use TOSSIM simulation to verify our model in Section III-A, after which the camera FOV overlap model, the multi-hop communication model as well as the model for constructing vision graph in WMSNs are analyzed in Section III-B. The parameters used in the analysis are listed in Table II.

A. Model Verification

To verify the models, we have simulated the WMSNs using TOSSIM. In the simulations, 1000 network topologies are generated by Poisson process, and the size of the field is 50 m \( \times \) 50 m. In each topology, two cameras nodes are setted with random distance and random direction. The vision angle of each camera is \( \frac{\pi}{18} \). Note all the camera pairs in the 1000 topologies are vision graph neighbors. The other parameters are set the same as in Table II. In Fig. 4, the theoretical and the simulation results of the probability of constructing the vision graph are shown.

It is observed from Fig. 4 that the mathematical models capture the characteristics of the FOV overlap and the multi-hop communications. For our settings, instead of unlimited flooding, 5-hop broadcast is sufficient to construct the vision graph. In other words, when deploying the WMSN, the camera nodes need to broadcast a hello message with maximum hop number of 5 to find the vision graph neighbors.

B. Model Analysis

In this section, the three models are analyzed. In Fig. 5, the probability of camera FOV overlap over distance is shown for
different camera vision angles. It is observed that when the camera vision angle is larger, the probability that the two cameras have overlapped FOVs is greater. At the distance of 5 m, if the camera vision angle is $\frac{\pi}{4}$, it is 12% more likely to have a vision graph neighbor than camera vision angle of $\frac{\pi}{2}$. Also, the decrease of the probability over distance is not constant. Around distance of 10 m, there is a sharp drop, which means most vision graph neighbors will be in a close distance.

The probability of connectivity for multi-hop communication is depicted in Fig. 6. Note in this figure, the hop numbers are the maximum allowed hop numbers. It is shown that if the 1-hop coverage is $r$, the coverage for $n$-hop is a little more than $nr$. For example, at 90% connectivity, the coverage of 1-hop is 4.57 m, however, the coverage of 3-hop is 14.73 m and the coverage of 5-hop is 26.39 m. Because the maximum distance of two vision graph neighbors in our setting is 20 m, we would assume the needed hop number is 4.

The probability of constructing the vision graph as a function of maximum hop numbers is shown in Fig. 7 for different camera vision angles. It is expected that because when the camera vision angle is greater, the two cameras are more likely to be vision graph neighbors, thus given a maximum hop number, the probability of constructing the vision graph is lower. However, the analysis shows the decrease of the probability is not significant. In fact, when the maximum hop number is 2, where the difference is most notable, the probability of constructing the vision graph for camera vision angle of $\frac{\pi}{4}$ is 52.08%, and for camera vision angle of $\frac{\pi}{3}$, the probability is 48.28%.

IV. Conclusions

In this paper, we address the problem of constructing vision graph in WMSNs. The mathematical model for camera FOV overlap is developed to analyze the probability of vision graph neighbor over distance. Meanwhile, multi-hop communication model with channel model in consideration is developed to analyze the probability of multi-hop coverage. Simulations are established to verify these models, which shows the mathematical models capture the characteristics of the FOV overlap and the multi-hop communications. For our settings, instead of unlimited flooding, 5-hop broadcast is sufficient to construct the vision graph.

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